

Quantum effects in radiation on short bunches

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Abstract

The radiation caused by particles of one bunch in the collective electromagnetic field of the short oncoming bunch is studied. Quantum effects are calculated for the spectrum of radiated photons. Using this spectrum, the dependence of the relative energy loss δ on a quantum parameter κ is discussed. It is shown that the behaviour of δ changes considerably with the increase of that parameter. In the classical regime ($\kappa \ll 1$) the energy loss is proportional to the incoming particle energy, while in the extreme quantum regime ($\kappa \gg 1$) the energy loss becomes a constant. The coherent e^+e^- pair production for γe colliders as cross-channel to CBS is considered.

1 Introduction

There are two types of coherent radiation at colliders — beamstrahlung (BS) and coherent bremsstrahlung (CBS). Beamstrahlung takes place on colliders with long bunches when the deflection angle of a radiating particle θ_d with Lorentz factor γ_e is much larger than the typical radiation angle $\theta_r \sim 1/\gamma_e$, it occurs mainly at e^+e^- linear colliders. CBS occurs at most of the existing colliders having short bunches ($\theta_d \ll \theta_r$). The different types of coherent radiation can be characterized by the parameter η which is related to these angles via¹

$$\frac{\theta_d}{\theta_r} \sim \eta = \frac{N_p r_e}{\sigma_x + \sigma_y} . \quad (1)$$

Classical and quantum regimes for BS ($\eta \gg 1$) are discussed in a number of papers (see, for example, the review [1]). For CBS ($\eta \ll 1$), only the classical regime has been considered up to now [2]-[6]. In our previous paper [6] we have presented a simple method for calculating CBS based on a developed equivalent photon approximation for coherent

¹Restricting ourselves to e^+e^- colliders, we consider, for definiteness, the photon radiation by electrons moving through a positron bunch. We denote by N_e and N_p the numbers of particles in the electron and positron bunches. σ_z is the longitudinal, σ_x and σ_y are the horizontal and vertical transverse sizes of the positron bunch, $\gamma_e = E_e/m_e c^2$ is the electron Lorentz factor and $r_e = e^2/m_e c^2$ is the electron classical radius.

processes. Here we apply this method to study the quantum effects for CBS. They are characterized by the quantum parameter κ which is defined by the ratio

$$\kappa = \frac{E_c}{E_e} , \quad (2)$$

where the critical energy E_c for CBS is given by the expression

$$E_c = \frac{4\gamma_e^2 \hbar c}{\sigma_z} . \quad (3)$$

The classical regime is characterized by $\kappa \ll 1$, the extreme quantum limit corresponds to $\kappa \gg 1$.

In Sect. 2 the spectrum of CBS photons is calculated as function of the photon energy E_γ and the quantum parameter κ . Additionally both quantum corrections to the classical case as well as corrections to the extreme quantum case are presented. Using the results of Sec. 2, the relative energy loss is discussed in Sect. 3. In the next section we discuss quantitatively a possibility to reduce the beamstrahlung energy loss using CBS bunchlets. Finally, we present in Sect. 5 a cross-channel to CBS – the coherent pair production at γe colliders. The number of produced e^+e^- pairs and the energy spectrum is calculated. Our main results are summarized in the Conclusions.

2 Spectrum of CBS

The energy spectrum of CBS photons is given in detail in [6]. Here, we summarize the results which are important for the following discussion. The number of CBS photons for a single collision of the bunches is

$$dN_\gamma = N_0 \Phi(E_\gamma/E_e, \kappa) \frac{dE_\gamma}{E_\gamma} . \quad (4)$$

The dimensionless constant N_0 is defined as

$$N_0 = \frac{8}{3} \alpha r_e^2 J(0) , \quad (5)$$

the function $J(\omega)$ can be found in [6]. For identical Gaussian beams the constant N_0 is well approximated by (see Appendix)

$$N_0 \approx 0.5 \alpha N_e \eta^2 . \quad (6)$$

The spectral function

$$\Phi(y, \kappa) = \frac{3}{2} \int_0^\infty \frac{dz}{(1+z)^2} \left[\frac{1+z^2}{(1+z)^2} (1-y) + \frac{1}{2} y^2 \right] \frac{J(\omega)}{J(0)} \quad (7)$$

with

$$y = \frac{E_\gamma}{E_e} \quad (8)$$

is normalized by the condition

$$\Phi(0, \kappa) = 1 . \quad (9)$$

The integration variable z is related to the polar angle θ of CBS photons $z = (\gamma_e \theta)^2$. The energy $\hbar\omega$ appearing in $J(\omega)$ is defined by

$$\hbar\omega = (1+z) \frac{E_e}{4\gamma_e^2} \frac{y}{1-y} . \quad (10)$$

For the practically important case of Gaussian beams one has

$$\frac{J(\omega)}{J(0)} = \exp \left[- \left(\frac{\omega \sigma_z}{c} \right)^2 \right] = \exp \left[- \left(\frac{1+z}{\kappa} \frac{y}{1-y} \right)^2 \right] . \quad (11)$$

In this case the spectral function simplifies to

$$\Phi(y, \kappa) = (1-y) \Phi_1(u) + \frac{3}{4} y^2 \Phi_2(u) , \quad u = \frac{1}{\kappa} \frac{y}{1-y} \quad (12)$$

where the functions $\Phi_i(u)$ are defined as

$$\Phi_1(u) = \frac{3}{2} \int_0^\infty \frac{1+z^2}{(1+z)^4} \exp \left[-(1+z)^2 u^2 \right] dz , \quad (13)$$

$$\Phi_2(u) = \int_0^\infty \frac{1}{(1+z)^2} \exp \left[-(1+z)^2 u^2 \right] dz . \quad (14)$$

The expansions of $\Phi_i(u)$ for small and large u are given by

$$\Phi_1(u) = \begin{cases} 1 - \frac{3}{2} \sqrt{\pi} u , & u \ll 1 \\ \frac{3}{4u^2} \left(1 - \frac{5}{2u^2} + \frac{37}{4u^4} + \dots \right) \exp(-u^2) , & u \gg 1 \end{cases} \quad (15)$$

and

$$\Phi_2(u) = \begin{cases} 1 - \sqrt{\pi} u , & u \ll 1 \\ \frac{1}{2u^2} \left(1 - \frac{3}{2u^2} + \frac{15}{4u^4} + \dots \right) \exp(-u^2) , & u \gg 1 . \end{cases} \quad (16)$$

To see the transition from the classical to the quantum regime we show in Fig. 1 the spectrum of CBS photons for different values of the parameter κ . In the classical case ($\kappa \ll 1$) the spectral function depends on the ratio y/κ only

$$\Phi(y, \kappa) = \Phi_1(y/\kappa) = \Phi_1(E_\gamma/E_c) , \quad (17)$$

and photons with energies larger than E_c practically do not contribute to the distribution. With increasing κ the fraction of high energy photons increases.

In the extreme quantum regime the quantity $\omega \sigma_z / c$ (see (11)) is much smaller than 1 for almost all values of E_γ , excluding the region of E_γ close to E_e . Therefore, the ratio $J(\omega)/J(0)$ can be replaced by one and the spectral function (at $1-y \gg 1/\kappa$) simplifies to

$$\Phi(y, \kappa) = 1 - y + \frac{3}{4} y^2 . \quad (18)$$

In other words, the positron bunch acts as a particle without inner structure, and the spectral function does not depend on the details of the bunch densities. It is clear that this function coincides with the spectral function of electron radiation on a point-like particle.

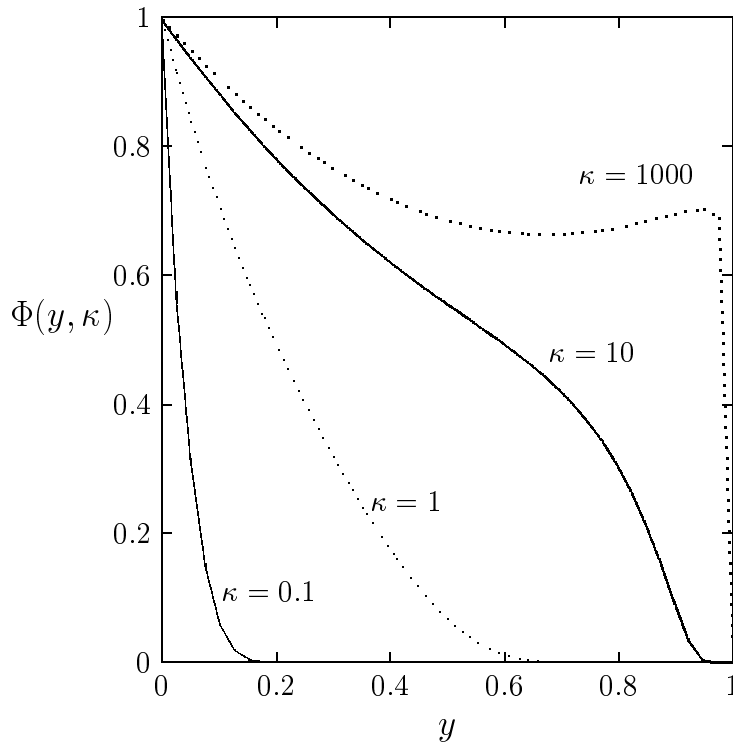


Figure 1: Spectral function $\Phi(y, \kappa)$ (see (12)) as function of the energy fraction $y = E_\gamma/E_e$ for different quantum parameters κ

Remind that the standard cross section for the electron radiation in the incoherent process $e^-e^+ \rightarrow e^-e^+\gamma$ is (see [7], §97)

$$d\sigma = \frac{16}{3}\alpha r_e^2 \left(1 - y + \frac{3}{4}y^2\right) \frac{dy}{y} \left(\ln \frac{4E_p E_e(1-y)}{m_e^2 c^4 y} - \frac{1}{2}\right) . \quad (19)$$

Eq. (19) contains the same spectral function as (18).

This universal dependence on y is violated only near the kinematical boundary $y \approx 1$ where

$$1 - y < \frac{1}{\kappa} \ll 1 . \quad (20)$$

For these photon energies the spectrum decreases very sharply (see Fig. 1) and the shape depends for Gaussian beams only on the longitudinal size of the positron bunch (in general on its density)

$$\Phi(y, \kappa) = \frac{3}{8u^2} \exp(-u^2) , \quad u \gg 1 . \quad (21)$$

3 Relative energy loss

The knowledge and the control of energy losses is one of the important collider physics problems.

We define the relative energy loss as follows

$$\delta(\kappa) = \frac{1}{E_e N_e} \int E_\gamma dN_\gamma = \frac{N_0}{N_e} \int_0^1 \Phi(y, \kappa) dy . \quad (22)$$

Introducing the transformation

$$x = \frac{y}{1-y} (1+z) \quad (23)$$

the integration over z can be performed explicitly and the energy loss is found in the form

$$\delta(\kappa) = \frac{N_0}{N_e} \int_0^\infty f(x) \frac{J(\omega)}{J(0)} dx, \quad \omega = x \frac{E_e}{4\gamma_e^2 \hbar} \quad (24)$$

where

$$f(x) = \frac{3(x^2 - 4x - 12)}{4x^4} \ln(x+1) + \frac{9}{x^3} - \frac{3(x+2)}{4(x+1)x^2} + \frac{x+3}{8(x+1)^3} . \quad (25)$$

The main contribution to the energy loss arises from the integration region

$$\frac{\omega \sigma_z}{c} = \frac{x}{\kappa} \sim \frac{y}{\kappa(1-y)} \lesssim 1 \quad (26)$$

where the ratio $J(\omega)/J(0)$ (see (11)) is of the order of one. Indeed, the region of small x does not contribute significantly to δ since the function $f(x)$ can be approximated for $x \ll 1$ by

$$f(x) = \frac{1}{2} - \frac{21}{20} x + \dots . \quad (27)$$

The region of large x is suppressed due to the behaviour of $J(\omega)/J(0)$ and $f(x)$.

From the estimate (26) it follows that small photon energies $E_\gamma \lesssim E_c$ dominate the energy loss at $E_c \ll E_e$, while at $E_c \gg E_e$ all energies E_γ are important up to the maximal value $E_\gamma \approx E_e$.

For Gaussian beams Eq. (24) simplifies to

$$\delta(\kappa) = \frac{N_0}{N_e} G(\kappa), \quad G(\kappa) = \int_0^\infty f(x) \exp\left(-\frac{x^2}{\kappa^2}\right) dx . \quad (28)$$

Its expansion at small and large values of parameter κ is found to be

$$\delta = \begin{cases} \delta^{\text{class}} \left(1 - \frac{21}{10\sqrt{\pi}} \kappa + \dots\right), & \kappa \ll 1 \\ \frac{3}{4} \frac{N_0}{N_e} \left[1 - \frac{\sqrt{\pi}}{\kappa} \left(\ln \frac{\kappa}{2} + \frac{1}{6} - \frac{\gamma_E}{2}\right)\right], & \kappa \gg 1 . \end{cases} \quad (29)$$

Here the energy loss in the classical limit is

$$\delta^{\text{class}} = \frac{\sqrt{\pi}}{4} \frac{N_0}{N_e} \kappa , \quad (30)$$

and $\gamma_E = 0.5772$ denotes the Euler constant.

The energy loss as function of the parameter κ is presented in Fig. 2 and in Table 1. For identical beams, the classical energy loss is well approximated by

$$\delta^{\text{class}} \approx 0.22 \alpha \eta^2 \kappa . \quad (31)$$

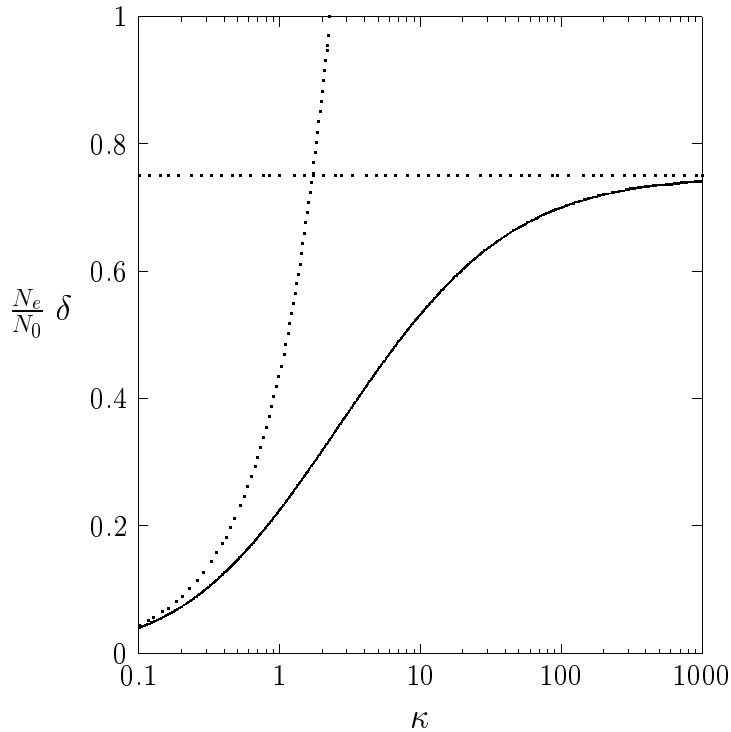


Figure 2: Normalized energy loss $(N_e/N_0)\delta$ as function of the quantum parameter κ (solid line), the classical and quantum limits are indicated by dotted lines

Table 1: Few values of the function $G(\kappa)$ from Eq. (28)

κ	0.1	0.2	0.5	0.7	1.0	2.0	5.0	7.0	10.0	50.0	100.0
$G(\kappa)$	0.039	0.073	0.145	0.18	0.23	0.32	0.45	0.49	0.53	0.67	0.70

We note that the energy loss for the coherent bremsstrahlung δ becomes constant at very large values of the quantum parameter κ (large beam energies).

It is interesting to compare the result derived for the energy loss in CBS ($\eta \ll 1$) with that of the beamstrahlung ($\eta \gg 1$). Since in the classical limit the energy loss is determined by the square of the electromagnetic fields, the formula is the same for both cases (see Eq. (30)). The same argument holds if one studies the classical energy loss as function of the impact parameter \mathbf{R} between the two colliding bunch axes. The whole effect is determined by the \mathbf{R} dependence of N_0 which has been described in detail in [6, 4], for flat and round bunches. Note that an increase of the energy loss is expected for non head-on collisions.

For beamstrahlung, the commonly used quantum parameter Υ is the ratio of the average critical energy for beamstrahlung $\langle E_c^{BS} \rangle$ to the electron energy E_e [1]. It can be

expressed through the parameters κ and η (for identical beams)

$$\Upsilon = \frac{2 \langle E_c^{BS} \rangle}{3 E_e} = \frac{5}{24} \eta \kappa . \quad (32)$$

An approximate expression for the BS energy loss is given in [1]

$$\delta^{BS} = \delta^{\text{class}} \left(1 + (1.5\Upsilon)^{2/3} \right)^{-2} . \quad (33)$$

Comparing the loss in the extreme quantum cases we note the different dependence on the incoming electron energy ($\kappa \propto E_e$)

$$\begin{aligned} \frac{\delta}{\delta^{\text{class}}} &\rightarrow \frac{3}{\sqrt{\pi\kappa}} , \\ \frac{\delta^{BS}}{\delta^{\text{class}}} &\rightarrow \frac{0.58}{\Upsilon^{4/3}} . \end{aligned} \quad (34)$$

The parameter Υ is very small for most of the proposed linear colliders. Therefore, the beamstrahlung is mainly classical allowing to transform some known CBS properties of the energy loss to colliders with large η .

Let us consider, for example, the TESLA collider [8] with flat transverse beams $\sigma_x/\sigma_y = 600 \text{ nm}/6.5 \text{ nm} \approx 100$, $\sigma_z = 0.5 \text{ mm}$, the design beam energy $E_e = 500 \text{ GeV}$ with $N_e = N_p = 1.8 \times 10^{10}$ particles per bunch. The BS quantum parameter Υ is 0.053 leading to $\delta = 0.024$ which is about 30 % smaller than the classical energy loss. In this case the dependence of the energy loss on the vertical beam displacement R_y should be qualitatively the same as that for the $DA\Phi NE$ collider with approximately the same transverse beam size ratio. For that collider the number of produced photons and, therefore, the energy loss increases almost two times at $R_y = 4\sigma_y$ (see [6]).

4 Remark on the problem of beamstrahlung reduction using CBS bunchlets

A few years ago an idea has been proposed by Chen [9] to reduce the BS energy loss. The discussion in [9] was on a rough qualitative level only. In our terminology this proposal can be formulated as follows: partitioning a particle bunch into a train of bunchlets, one could change the nature of radiation from BS to CBS. This may not be a very promising idea since such a proposal leads to a lot of difficulties including an increase of the total bunch length. If we omit such problems, we are able to discuss this idea quantitatively. In what follows we also neglect, for simplicity, disruption effects.

We notice that beamstrahlung and CBS depend differently on the number of bunchlets n_b . Denoting by the index i the contribution of an individual bunchlet i we see from the above formulae that the ratio $\delta_i^{BS}/\delta_i^{\text{class}}$ depends on the parameter $\Upsilon_i \propto N_i/\sigma_{zi}$ only. Neglecting the longitudinal boundary effects, Υ_i is the same as the quantum parameter $\Upsilon \propto N/\sigma_z$ of the whole bunch. On the contrary, the parameters η and κ of the whole bunch change to

$$\eta_i = \frac{\eta}{n_b} , \quad \kappa_i = n_b \kappa . \quad (35)$$

As long as the number of bunchlets is such that $\eta_i \gg 1$, one remains in the beamstrahlung regime and practically no energy reduction is obtained.

If η_i is smaller than one, we arrive at the CBS regime assuming that the spacing between two subsequent bunchlets is such that their radiation is incoherent. In that case the ratio of the energy losses $r_i = \delta_i^{CBS}/\delta_i^{class}$ depends on κ_i and, therefore, on n_b . The ratio r_i decreases with increasing n_b (remind that $r_i \rightarrow 2.3/\kappa_i \propto 1/n_b$ at $\kappa_i \gg 1$). Therefore, one can in principle obtain a considerable energy loss reduction for large enough number of bunchlets. To be precise, the following conditions for a decreasing energy loss should be fulfilled. First of all, the parameter $\eta_i = \eta/n_b$ should be smaller than one, hence, the number of bunchlets n_b should be greater than η . Additionally, to get a *considerable* reduction one should use such values of the parameter $\kappa_i = n_b \kappa \gtrsim 1$ which leads to $n_b \gtrsim 1/\kappa$. As a result, we obtain the requirement

$$n_b \gtrsim \max\left(\frac{1}{\kappa}, \eta\right). \quad (36)$$

As a side remark we notice, that by choosing a relatively large value of n_b one could reach the situation where the length of a bunchlet $\sigma_{zi} = \sigma_z/n_b$ can become smaller than its transverse size σ_x . In this case we are still in the validity range of our formulae for CBS (where we used the restriction $\gamma_e \sigma_{zi} \gg \sigma_x$ only), though it may be technically difficult to realize.

Let us give a numerical example to show how the discussed reduction works. We consider the TESLA accelerator mentioned above. For this collider we have $\eta = 85$ and $\kappa = 0.003$. The minimal number of bunches according to (36) is 330. For this value one obtains $\eta_i = 0.26$ and $\kappa_i = 1$ which leads to an reduction of the original energy loss by a factor 1.4. Increasing the used number of bunches three times we get an reduction of the energy loss by the factor 2.5 (notice that in this case $\sigma_{zi} = \sigma_x$).

5 Coherent pair production

In this section we consider a cross-channel to CBS – the coherent pair production at γe colliders. This process can be considered as e^+e^- pair production in collisions of initial photons of the energy E_γ with the equivalent photons of the energy $\hbar\omega$ corresponding to the collective field of the electron bunch (see Fig. 3). The number of e^+e^- pairs produced

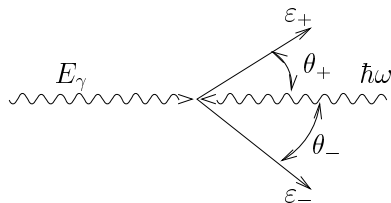


Figure 3: Kinematics for the process $\gamma\gamma \rightarrow e^+e^-$, ε_{\mp} denote the energies and θ_{\mp} are the polar angles of the electron/positron of the produced pair

per single bunch crossing is

$$dN_{e^+e^-} = dL_{\gamma\gamma}(\omega) d\sigma_{\gamma\gamma}(\omega, E_\gamma) , \quad (37)$$

where the spectral luminosity of the $\gamma\gamma$ collisions is equal to (compare Eqs. (39-41) of [6])

$$\begin{aligned} dL_{\gamma\gamma}(\omega) &= \frac{\alpha}{\pi} \frac{d\omega}{\omega} J(\omega) , \\ J(\omega) &= 4\pi \int \frac{\mathbf{q}_\perp}{\mathbf{q}_\perp^2} \frac{\mathbf{q}'_\perp}{\mathbf{q}'_\perp^2} F_e(\mathbf{q}) F_e^*(\mathbf{q}') F_\gamma(\mathbf{q} - \mathbf{q}') \frac{d^2 q_\perp d^2 q'_\perp}{(2\pi)^4} . \end{aligned} \quad (38)$$

Here F_e and F_γ are the form factors of the electron and the initial photon bunches, respectively, $d\sigma_{\gamma\gamma}$ is the cross section for the $\gamma\gamma \rightarrow e^+e^-$ process.

Using the total pair production cross section as function of the variable v (the ratio of the invariant mass squared to the threshold energy squared)

$$v = \frac{\hbar\omega E_\gamma}{m_e^2 c^4} \quad (39)$$

we obtain the total number of produced pairs

$$N_{e^+e^-} = N_0 \int_1^\infty \frac{\sigma_{\gamma\gamma}(v)}{\sigma_0} \frac{J(\omega)}{J(0)} \frac{dv}{v} . \quad (40)$$

The constant N_0 is defined in (5), $\sigma_0 = (8\pi/3)r_e^2$ is the Thompson cross section. Choosing the main spectral component (see (11)) $\omega = c/\sigma_z$ and $v \approx 1$, one finds a characteristic energy E_{char} for the produced lepton pair

$$E_{\text{char}} = \frac{m_e^2 c^3}{\hbar} \sigma_z . \quad (41)$$

For photon energies $E_\gamma > E_{\text{char}}$ the pair production becomes important.

Introducing the ratio

$$\tau = E_\gamma / E_{\text{char}} \quad (42)$$

one gets for Gaussian beams ($\omega\sigma_z/c = v/\tau$, σ_z is the longitudinal electron bunch size)

$$N_{e^+e^-}(\tau) = N_0 \int_1^\infty \phi(v) \exp\left(-\frac{v^2}{\tau^2}\right) dv \quad (43)$$

with the function (comp. [7], §88)

$$\begin{aligned} \phi(v) &= \frac{\sigma_{\gamma\gamma}(v)}{\sigma_0} \frac{1}{v} \\ &= \frac{3}{8v^4} \left[2(v^2 + v - \frac{1}{2}) \ln(\sqrt{v} + \sqrt{v-1}) - (v+1)\sqrt{v^2-v} \right] . \end{aligned} \quad (44)$$

It is not difficult to find that

$$N_{e^+e^-}(\tau) = N_0 \begin{cases} \frac{7}{12} , & \tau \gg 1 \\ \frac{3}{64} \sqrt{2\pi} \tau^3 e^{-1/\tau^2} , & \tau \ll 1 . \end{cases} \quad (45)$$

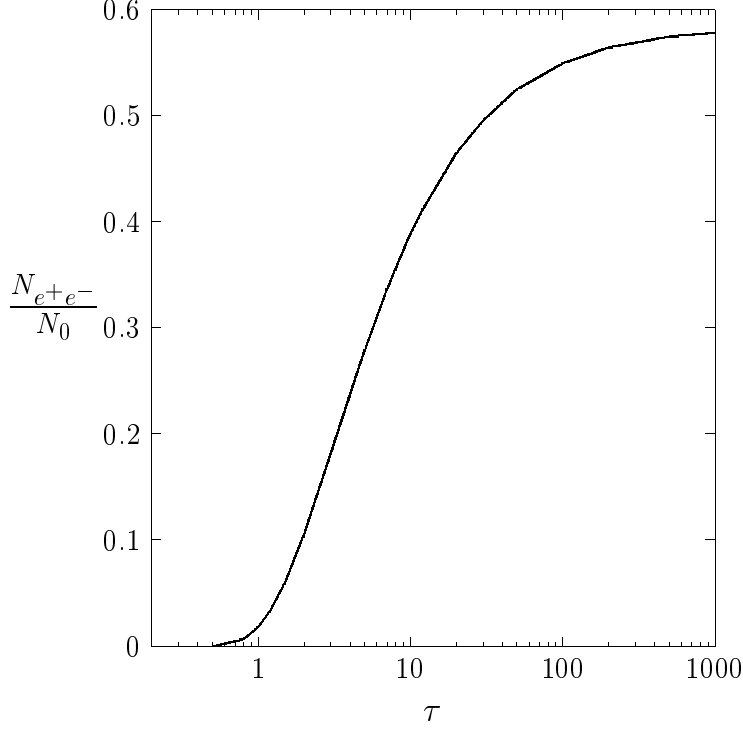


Figure 4: Normalized total number of produced pairs $N_{e^+e^-}/N_0$ as function of the parameter τ (42)

The normalized number of produced pairs as the function of the ratio τ is shown in Fig. 4. As expected a rapid increase of the pair creation rate is observed for $\tau \gg 1$.

Taking the differential cross section $d\sigma_{\gamma\gamma}(\omega, E_\gamma, \varepsilon_\pm, \mathbf{p}_\pm)$ from [10] we obtain the energy-angular distribution of the produced pairs (ε_\pm and \mathbf{p}_\pm are the energy and momentum of the produced e^\pm , respectively)

$$dN_{e^+e^-}(\tau) = N_0 \frac{dzdx}{(1+z)^2} \left[\frac{3}{4} - \frac{3}{2} x(1-x) \frac{1+z^2}{(1+z)^2} \right] \frac{J(\omega)}{J(0)} \quad (46)$$

where

$$x = \frac{\varepsilon_+}{E_\gamma}, \quad \varepsilon_- = (1-x)E_\gamma, \quad z = \frac{\mathbf{p}_{+\perp}^2}{(m_e c)^2}, \quad \hbar\omega = \frac{m_e^2 c^4}{4E_\gamma} \frac{1+z}{x(1-x)}. \quad (47)$$

It follows from Eq. (46) that the main contribution to the pair production is given by the region $z \lesssim 1$, i.e. $|\mathbf{p}_{+\perp}| \lesssim m_e c$.

Finally, integrating Eq. (46) over z , we obtain the energy spectrum of produced e^+ for Gaussian beams

$$\frac{dN_{e^+}(x, \tau)}{dx} = N_0 \left[\frac{3}{4} \Phi_2(u) - x(1-x) \Phi_1(u) \right], \quad u = \frac{1}{\tau} \frac{1}{4x(1-x)} \quad (48)$$

where the functions $\Phi_1(u)$ and $\Phi_2(u)$ are defined by Eqs. (24) and (25). The corresponding spectra for different values of the parameter τ (normalized at the energy fraction $x = 0.5$) are shown in Fig. 5.

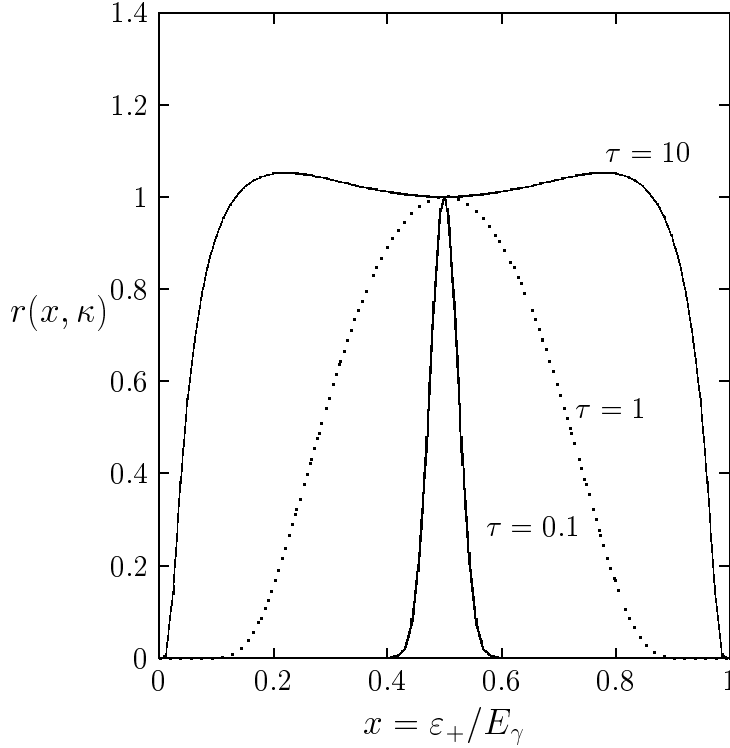


Figure 5: Energy spectrum $r(x, \tau) = (dN_{e^+}(x, \tau)/dx)/(dN_{e^+}(x, \tau)/dx)|_{x=0.5}$ for different τ values

For small values of u ($u \ll 1$) the spectrum can be approximated by

$$\frac{dN_{e^+}(x, \tau)}{dx} = N_0 \left[\frac{3}{4} - x(1-x) \right]. \quad (49)$$

In this limit, the energy spectrum becomes similar to the well-known distribution of e^+e^- pairs for the photoproduction off nuclei (see [7], §94). For large values of u ($u \gg 1$) the spectrum is suppressed like

$$\frac{dN_{e^+}(x, \tau)}{dx} = N_0 \frac{3}{8} [1 - 2x(1-x)] \frac{1}{u^2} \exp(-u^2). \quad (50)$$

For $\tau \ll 1$ it follows from (48,50) that the produced electrons and positrons have approximately equal energies, $\varepsilon_+ \approx \varepsilon_-$. Additionally, taking into account $\mathbf{p}_{+\perp} = -\mathbf{p}_{-\perp}$, their polar angles are approximately equal $\theta_+ \approx \theta_-$.

6 Conclusions

In the present paper we have calculated quantum effects of coherent bremsstrahlung at colliders with short bunches. The calculation is based on the earlier developed equivalent photon approximation for coherent processes.

The spectrum and the energy loss are found as function of the quantum parameter κ (2). The classical regime is given by $\kappa \ll 1$, the extreme quantum limit corresponds to

$\kappa \gg 1$. Quantum corrections to the classical limit as well as corrections to the extreme quantum cases are found. It was shown that the electron energy loss changes its behaviour with the growth of the parameter $\kappa \propto E_e$. In the classical case we have $\delta \propto \kappa$ while in the extreme quantum regime δ becomes a constant.

As an example, the obtained formulae have been used to discuss quantitatively the proposal [9] to reduce the beamstrahlung energy loss using CBS bunchlets. The quantum effects in CBS may be also important for linear superconductive colliders recently under discussion. For these colliders it is planned to use the bunches of particles not only once but repeatedly which leads to the requirement of small deflection angles.

Finally, the coherent e^+e^- pair production for γe colliders with short bunches is discussed. The total number of pairs is calculated, the energy spectrum and the energy-angular distribution are presented.

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A Appendix: Approximation of constant N_0

The dimensionless constant N_0 (5) depends on the transverse densities of the electron and positron bunches. In the case of Gaussian beams this quantity can be calculated in a form of a one-dimensional integral [4]. For identical beams the limits of round ($\sigma_x = \sigma_y$) and flat ($\sigma_y \ll \sigma_x$) beams are known exactly [4, 6]

$$N_0^{\text{round}} = c^{\text{round}} \alpha N_e \eta^2, \quad c^{\text{round}} = \frac{16}{3\pi} \ln \frac{4}{3} \approx 0.4884 \quad (51)$$

and

$$N_0^{\text{flat}} = c^{\text{flat}} \alpha N_e \eta^2, \quad c^{\text{flat}} = \frac{8}{9\sqrt{3}} \approx 0.5132 \quad (52)$$

where η is given by (1). In the estimate (6) we have assumed

$$c^{\text{round}} \approx c^{\text{flat}} \approx 0.5. \quad (53)$$

A simple interpolation formula can be given parametrizing the dependence of $N_0/(\alpha N_e \eta^2)$ for identical Gaussian beams on the ratio σ_x/σ_y (independent on the other bunch parameters) within 0.11 per cent accuracy

$$\frac{N_0}{\alpha N_e \eta^2} = c^{\text{round}} + (c^{\text{flat}} - c^{\text{round}}) \frac{2}{\pi} \arctan \left[0.191 \left(\frac{\sigma_x}{\sigma_y} - 1 \right) \right]. \quad (54)$$

In Fig. 6 we show the exact behaviour of $N_0/(\alpha N_e \eta^2)$ (full line) together with the presented approximation (dotted line).

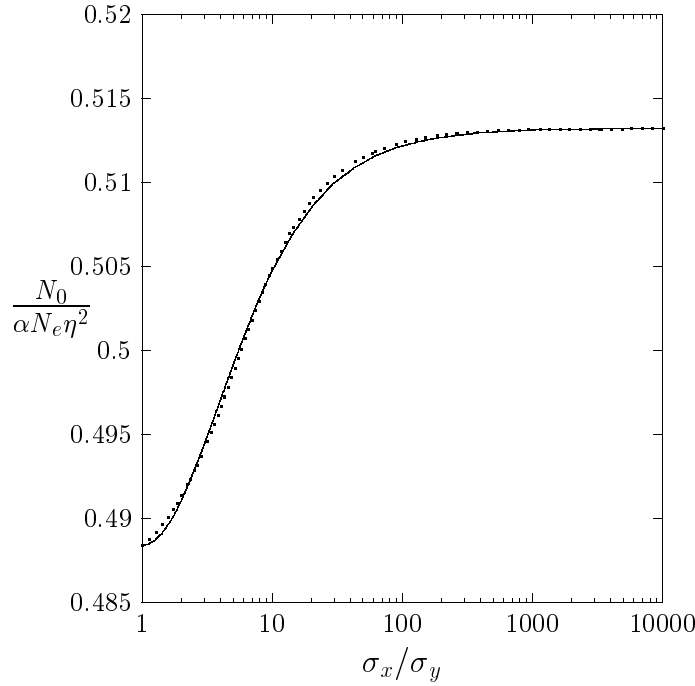


Figure 6: Comparison of exact (solid line) and approximate (dotted line) behaviour of $N_0/(\alpha N_e \eta^2)$ as function of σ_x/σ_y for identical beams

References

- [1] P. Chen: in "An Introduction to Beamstrahlung and Disruption", ed. by M. Month and S. Turner, Lectures Notes in Physics 296, Berlin, Heidelberg, New York: Springer 1988.
- [2] M. Bassetti, J. Bosser, M. Gygi-Hanney, A. Hoffmann, E. Keil and R. Coisson: IEEE Trans. on Nucl. Science NS-30 (1983) 2182
- [3] I.F. Ginzburg, G.L. Kotkin, S.I. Polityko and V.G. Serbo: Phys. Rev. Lett. 68 (1992) 788; Phys. Lett. B286 (1992) 392; Z.. Phys. C60 (1993) 737
- [4] I.F. Ginzburg, G.L. Kotkin, S.I. Polityko and V.G. Serbo: Yad. Fis. (in Russian) 55 (1992) 3310 and 3324.
- [5] S.I. Polityko and V.G. Serbo: Phys. Rev. E51 (1995) 2493.
- [6] R. Engel, A. Schiller, V.G. Serbo: Leipzig University Preprint UL 95/29; Z. Phys. C in print
- [7] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii: Quantum Electrodynamics, Moscow: Nauka 1989.
- [8] R. Brinkmann: SBLC and TESLA general design overview, Presentation at the Linear Collider workshop, Gran Sasso, Italy, 1995.

- [9] P. Chen: SLAC Preprint SLAC-PUB-4293, 1987.
- [10] G.L. Kotkin, V.G. Serbo and A. Schiller: Int. J. Mod. Phys. A7 (1992) 4707.